Maximum Reinforcement and Ductility in Concrete Masonry Shear Walls

Introduction

Concrete masonry shear walls must possess sufficient strength to resist axial, flexural, and shear demands. Additionally, these walls must satisfy specific detailing requirements as outlined in the prevailing codes. These requirements are to ensure that the walls possess adequate ductility in the event of seismic activity. The use of maximum reinforcement ratios is intended to provide ductility by ensuring that the tensile reinforcement yields prior to crushing of the compressive zone.

Examples will be provided to illustrate how to determine the maximum reinforcement permitted in concrete masonry walls in accordance with the 2006 International Building Code (IBC) [1]. For masonry design, the IBC references the ACI 530-05/ASCE 5-05/TMS 402-05 [2], which is also referred to as the 2005 Masonry Standards Joint Committee Building Code (MSJC).

Nomenclature

The nomenclature used in this article is as follows:

- \( a \) = Depth of equivalent rectangular stress block
- \( A_n \) = Net area of wall
- \( A_{sb} \) = Area of steel in balanced condition
- \( A_{s,max} \) = Maximum area of steel allowed
- \( b \) = Nominal width of wall
- \( c \) = Distance from extreme compression fiber to neutral axis
- \( C \) = Compression force
- \( C_m \) = Compression force in masonry
- \( C_s \) = Compression force in steel
- \( d \) = Distance from centroid of tensile reinforcement to extreme compression fiber
- \( E_m \) = Modulus of Elasticity of masonry
- \( E_s \) = Modulus of Elasticity of steel
- \( f' \) = Specified compressive strength of masonry
- \( f_b \) = Calculated compressive stress in masonry from flexure
- \( f_s \) = Calculated tensile stress in reinforcement
- \( f_y \) = Specified strength of steel reinforcement
- \( F_b \) = Allowable compressive stress due to flexure
- \( F_s \) = Allowable tensile or compressive stress in reinforcement
- \( j_b \) = Ratio of distance between centroid of flexural compressive forces and centroid of tensile forces at the balanced condition
Table 1 – Minimum Tension Reinforcement Strain Factor $\alpha$ at Ultimate Limit State for Concrete Masonry Members

<table>
<thead>
<tr>
<th>Basic Seismic Force Resisting System</th>
<th>$R$</th>
<th>$\frac{M_v}{V_{d_e}}$</th>
<th>$\alpha_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Special Reinforced Masonry Shear Walls</td>
<td>5</td>
<td>$\geq 1.0$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&lt; 1.0$</td>
<td>1.5</td>
</tr>
<tr>
<td>Intermediate Reinforced Masonry Shear walls</td>
<td>3.5</td>
<td>$\geq 1.0$</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&lt; 1.0$</td>
<td>1.5</td>
</tr>
<tr>
<td>Walls Loaded Out-of-Plane</td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>All Others</td>
<td>All</td>
<td>$\geq 1.0$</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$&gt; 1.5$</td>
<td>$&lt; 1.0$</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$&lt; 1.5$</td>
<td>$&lt; 1.0$</td>
<td>None</td>
</tr>
</tbody>
</table>

Figure 2 illustrates the calculations that need to be performed to determine if the reinforcement ratio is acceptable. The neutral axis is selected so that the masonry wall cross-section is in equilibrium with the axial load, which is determined using the load combination $D + 0.75L + 0.525Q_E$ (MSJC Section 3.3.3.5.1), where $Q_E$ is the effect of the horizontal component of the earthquake load. Note that reinforcement in compression may be used in evaluating equilibrium of the cross-section even if it is not laterally supported by ties (MSJC Section 3.3.3.5.1). Once the location of the neutral axis has been determined, the strain in the extreme tension reinforcement can be obtained and compared to the acceptable value of $\alpha \varepsilon_y$. If the maximum tensile strain in the steel is less than $\alpha \varepsilon_y$, the wall is over-reinforced and the amount of steel must be reduced or the wall redesigned.

Figure 2 – Maximum Reinforcement in Masonry Shear Walls

2006 IBC / 2005 MSJC Strength Design Requirements for Maximum Reinforcement

Figure 1 illustrates the relationship between steel tensile strain and the ductility or displacement capacity of a wall. As shown in the figure, walls with larger strains in the tension steel at the ultimate limit state have a greater displacement capacity.

The MSJC code evaluates the ductility of a wall using the tension reinforcement strain factor $\alpha$. Table 1 summarizes the MSJC strength design requirements for ductility capacity of various members using the tension reinforcement strain factor. Special reinforced and intermediate reinforced masonry walls, which are designed with higher values of $R$, are more likely to exhibit significant nonlinear response during design level earthquakes. These walls must be able to attain higher values of $\alpha$ at the ultimate limit state. Members designed with lower values of $R$ have less stringent requirements. Members with low aspect ratios, which deflect primarily as a result of shear deformation, are not likely to develop flexural plastic hinges that require significant ductility. Consequently, the code requirements are also less stringent for these members.
To simplify the calculations for maximum reinforcement, the steel reinforcement can be assumed to be evenly distributed as shown in Figure 3.

From Figure 3, the depth of the neutral axis is given by:

$$c = \frac{\varepsilon_{mu} L_w}{\alpha \varepsilon_y + \varepsilon_{mu}}$$  \hspace{1cm} (6)

The maximum permissible amount of reinforcement is given by:

$$A_{s,\text{max}} = \frac{0.64 f_m' b \left( \frac{\varepsilon_{mu} L_w}{\alpha \varepsilon_y + \varepsilon_{mu}} \right) - P}{f_y \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\alpha \varepsilon_y + \varepsilon_{mu}} \right)}$$  \hspace{1cm} (7)

Therefore, the maximum steel per unit length of wall is equal to:

$$\frac{A_{s,\text{max}}}{L_w} = \frac{0.64 f_m' b \left( \frac{\varepsilon_{mu} L_w}{\alpha \varepsilon_y + \varepsilon_{mu}} \right) - P}{f_y \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\alpha \varepsilon_y + \varepsilon_{mu}} \right)}$$  \hspace{1cm} (8)

In terms of the reinforcement ratio on the gross cross section:

$$\rho_{\text{max}} = \frac{A_{s,\text{max}}}{b L_w} = \frac{0.64 f_m' b \left( \frac{\varepsilon_{mu} L_w}{\alpha \varepsilon_y + \varepsilon_{mu}} \right) - P}{f_y \left( \frac{\alpha \varepsilon_y - \varepsilon_{mu}}{\alpha \varepsilon_y + \varepsilon_{mu}} \right) L_w b f_m'}$$  \hspace{1cm} (9)

Figure 4 shows the maximum reinforcement ratio for special reinforced masonry shear walls with various masonry compressive strengths. The figures illustrate the effect of axial load on ductility, and compares the maximum reinforcement ratio to the minimum reinforcement required in Seismic Design Category D and above. It is shown that the ductility requirements result in limits on the axial load that can be placed on a wall. Figure 5 shows the maximum reinforcement ratios for intermediate reinforced shear walls.

Note that Equations (7), (8), (9) and Figures 5 and 6 are only valid for shear walls with relatively closely spaced vertical reinforcement that is evenly distributed along the length of the wall. However, they do provide a tool for obtaining a quick estimate of the maximum reinforcement permitted in all walls.
Determine if the wall shown in Figure 6 satisfies the requirements of the maximum reinforcement ratio according to the 2006 IBC provisions for strength design. The fully grouted wall is constructed with 8-inch, medium-weight concrete masonry units (78 psf). The specified masonry compressive strength is 1500 psi and grade 60 steel is used. The vertical steel in the special reinforced masonry wall consists of #4 bars at 32 inches on center.

**Solution:**

The in-plane earthquake loads from the first floor are equal to:

\[ V_E = 52 + 30 + 18 = 100 \text{ kips} \]
\[ M_E = 52(34.5) + 30(24) + 18(13.5) = 2757 \text{ kip-ft} \]

Gravity loads at the first floor are equal to:

\[ P_D = \frac{22(3000 + 2500 + 2500)}{1000} + \frac{78(22)(34.5)}{1000} = 235 \text{ kips} \]
\[ P_L = \frac{22(1000 + 1000)}{1000} = 44 \text{ kips} \]
\[ P_{LR} = \frac{22(500)}{1000} = 11 \text{ kips} \]
According to MSJC Section 3.3.3.5.1d the maximum reinforcement must be checked using the axial load $P$, from the load combination $D+0.75L+0.525Q_E$:

$$P = D + 0.75L + 0.525Q_E$$
$$P = 235 + 0.75(44 + 11) + 0.525(100)$$
$$P = 328.75 \text{ kips}$$

For a special reinforced masonry wall, the tensile strain factor, $\alpha = 4$. Therefore using Equation (9):

$$\rho_{\text{max}} = \frac{0.64 \left( \frac{0.0025}{4(0.00207) + 0.0025} \right) - 328.75}{60 \left( \frac{0.0025}{4(0.00207) - 0.0025} \right) + 1.5 \left( \frac{0.0025}{4(0.00207) + 0.0025} \right)}$$
$$\rho_{\text{max}} = 0.0018$$

The use of #4 bars at 32 inches on center results in a reinforcing ratio of 0.00082. Since there is less steel in the wall than the maximum permitted, the code provision regarding the maximum reinforcing ratio is satisfied.

**2006 IBC / 2005 MSJC Allowable Stress Design Requirements for Maximum Reinforcement**

The MSJC provisions do not provide ductility-based limitations on the amount of reinforcement that can be placed in a wall. However, Section 2107.8 of the 2006 IBC limits the amount of flexural reinforcement in special reinforced masonry walls with a shear span ratio ($M/V_d$) equal to greater than 1.0 and an axial load greater than $0.5f'_m A_n$ to the following:

$$\rho_{\text{max}} = \frac{n f'_m}{2 f_y \left( n + \frac{f_y}{f'_m} \right)}$$

The maximum reinforcement ratio in Equation (10) does not apply to the out-of-plane design of shear walls.

Equation 1 is derived from the balanced stress condition. The section is considered to be balanced when the allowable masonry stress and allowable steel stress occur simultaneously.

Figure 7 – The Balanced Condition

Figure 7 shows a cross-section at the balanced condition. By similar triangles, the neutral axis is determined to be:

$$k_b = \frac{1}{1 + \frac{f'_m}{f_y}}$$

(11)

The moment arm at the balanced condition will be:

$$j_b = 1 - \frac{k_b}{\phi}$$

(12)

The moment on the cross section is:

$$M_b = \frac{1}{2} F_b j_b k_b b d^2$$

(13)

Solving for $d$, the effective depth of a cross section subjected to a given moment is:

$$d = \sqrt[2]{\frac{2 M_b}{F_b j_b k_b b}}$$

(14)

To maintain equilibrium in the cross section, the compressive forces in the masonry must be equal to the tensile force in the reinforcing steel:

$$C = T$$

(15)

$$\frac{1}{2} F_b k_b b d = F_s A_{sb}$$

(16)

Therefore, the reinforcement required to achieve the balanced condition is equal to:

$$A_{sb} = \frac{F_b k_b b d}{2 F_s}$$

(17)
or
\[
\rho_b = \frac{A_{sb}}{bd} = \frac{k_b F_b}{2F_s}
\]  

(18)

When we substitute the neutral axis depth, \( k_b \), from Equation (11) we obtain the following:
\[
\rho_b = \frac{n}{2 F_s \left( \frac{F_x}{F_b} \right)}
\]  

(19)

Substituting \( f_m' \) and \( f_y \) for \( F_b \) and \( F_s \) yields the maximum allowed reinforcement ratio as prescribed by the IBC (Section 2107.8):
\[
\rho_{max} = \frac{nf_m'}{2f_y \left( n + \frac{f_y}{f_m'} \right)}
\]

The above equation has severe limitations. It is derived for singly reinforced cross-sections and is therefore not completely appropriate for most shear walls, which have reinforcement distributed along the wall length. More importantly, the equation does not include the significant effect of axial load on the allowable maximum reinforcement, as shown in Figures 4 and 5. Figure 8 compares the strength design requirements with Equation (10) for walls with Grade 60 reinforcement. The figure shows that for masonry with a compressive strength of 1500 psi, the allowable stress design equation used in the IBC is non-conservative. For higher masonry compressive strengths Equation (10) is non-conservative for a wider range of axial loads.

To provide a more reasonable comparison with the strength design requirements, the following equation may be utilized for walls with Grade 60 steel:
\[
\rho_{max} = 5 \times 10^{-6} f_m' \left( 1 - \frac{P}{0.15 f_m' b d} \right)
\]  

(20)

Figure 9 provides a comparison of the suggested equation for masonry with a compressive strength of 1500 psi and 2500 psi.
Conclusion

Maximum reinforcement ratios and detailing requirements in the code seek to improve ductility by limiting the amount of steel used in masonry structures. The limited amount of steel ensures that the steel yields before the concrete masonry has a chance to fail in compression.

The 2006 IBC and 2005 MSJC strength design requirements provide limits on the amount of reinforcement in walls based on the maximum tensile strain in the reinforcing steel. Since the steel tensile strain at the ultimate limit state is directly related to displacement capacity, the strength design requirements provide a reasonable approach in determining the maximum amount of steel permitted.

While the MSJC requirements do not provide ductility-based limits on the amount of reinforcement that can be placed in shear walls under allowable stress design, the 2006 IBC includes an equation that is based on the balanced condition. The equation is generally non-conservative for shear walls, especially at higher axial loads. An alternative equation that compares favorably with the strength design requirements is suggested for use with allowable stress design.

For more information on the design of shear walls please see the 2006 edition of the “Design of Reinforced Masonry Structures”. This publication is published and made available through the Concrete Masonry Association of California and Nevada (CMACN).

References


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